Mark Scheme (Results)

Summer 2017

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM0) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers normally score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**

  If there is a wrong answer indicated always check the working in the body of the script and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread loses two A (or B) marks on that part, but can gain the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.
- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.
- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$, where $|pq|=|c|$ leading to $x=....$

$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required.
(Mark schemes may override this eg in a case of "prove or show...."

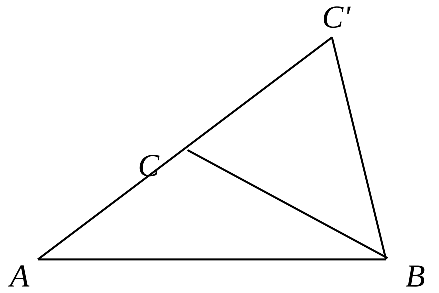
Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1	$16-\left(e^{x}\right)^{2}=6e^{x}$	M1
	$\left(e^{x}\right)^{2}+6e^{x}-16=0$	A1
	$\left(e^{x}+8\right)\left(e^{x}-2\right)=0$	M1
	$\left(e^{x}=-8 \quad \text{(not poss)}\right)$	
	$e^{x}=2 \quad x=\ln 2$	M1A1
		[5]
2	$V=\frac{1}{3}\pi r^{2}h=\frac{1}{9}\pi r^{3}$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{1}{3}\pi r^{2}$	M1
	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}r}{\mathrm{d}V}$	M1
	$=50\times\frac{3}{\pi\times10^{2}}$	dM1
	$=0.4774...=0.477$ cm/s	A1
		[5]
3 (a)	$\overrightarrow{CD}=\overrightarrow{CO}+\overrightarrow{OD}=\frac{2}{3}\mathbf{b}-\frac{1}{2}\mathbf{a}$	M1A1 (2)
(b)	$\overrightarrow{CE}=\overrightarrow{CO}+\overrightarrow{OE}=-\frac{1}{2}\mathbf{a}+2\mathbf{b}-\mathbf{a}=2\mathbf{b}-\frac{3}{2}\mathbf{a}$	M1A1 (2)
(c)	$\overrightarrow{CE}=2\mathbf{b}-\frac{3}{2}\mathbf{a}=3\left(\frac{2}{3}\mathbf{b}-\frac{1}{2}\mathbf{a}\right)=3\overrightarrow{CD}$	M1
	$\therefore$ C, D and E are collinear	A1 (2)
		[6]
	ALT: Use any other pair from $\overrightarrow{CD},\overrightarrow{DE},\overrightarrow{CE}$ M1 complete method; A1 all correct inc conclusion.	
4 (a)	$\tan\theta=3 \quad \tan\theta=-2$	M1
	$\theta=1.2490...=1.249$	A1
	$\theta=2.0344...=2.034$	A1 (3)

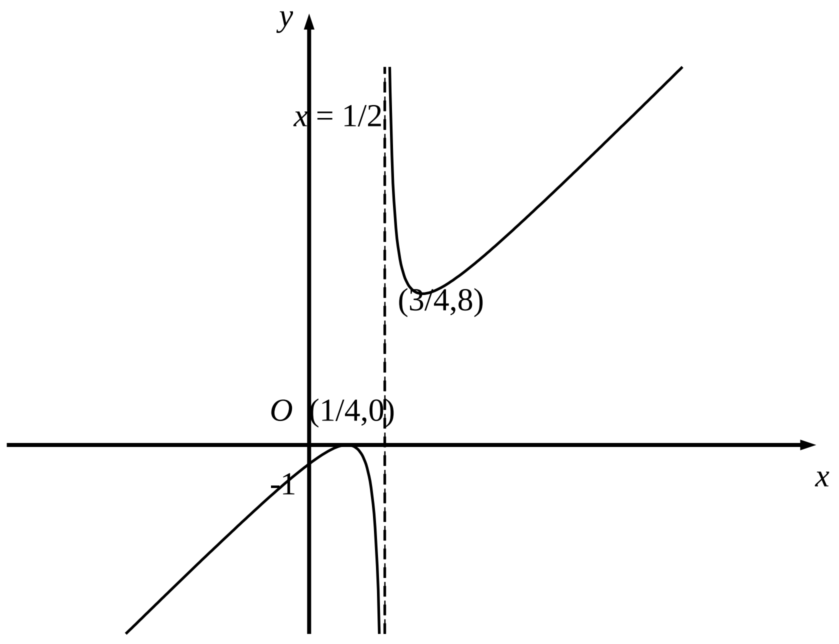
Question Number	Scheme	Marks
(b)	$6\left(1-\sin^2\theta\right)-\sin\theta=5$	M1
	$6\sin^2\theta+\sin\theta-1=0$	
	$\left(3\sin\theta-1\right)\left(2\sin\theta+1\right)=0$	M1
	$\sin\theta=\frac{1}{3}\quad \theta=0.33983...\ 2.8017...\quad \theta=0.3398,\ 2.802$	A1A1
		(4)
	$\left(\sin\theta=-\frac{1}{2}\quad \text{no solutions in range}\right)$	
		[7]
5(a)	$\frac{\sin 40}{7}=\frac{\sin C}{10}$	M1A1
	$C=\sin^{-1}\left(\frac{10\sin 40}{7}\right)=66.67...,113.32...$	
	$C=66.7°,\ 113.3°$	A1,A1 (4)
(b)		
	$\Delta CC'B$ is isosceles (seen explicitly or implied by next statement)	M1
	$CC'=2\times 7\cos 66.67$	M1A1ft
	$=5.543...$ cm = 5.54cm (Accept 5.54 (cm) or better)	A1 (4)
		[8]
	NB For longer methods: M1 trig statement; A1ft correct numbers, follow through angle found in (a); M1 complete to...A1 correct answer.	

Question Number	Scheme	Marks
6 (a)	$a+ar^2=250$	M1
	$ar+ar^2=150$	A1
	$\frac{1+r^2}{r+r^2}=\frac{5}{3}$	
	$2r^2+5r-3=0$	M1
	$(2r-1)(r+3)=0 \quad r=\frac{1}{2},\ r=-3$	M1A1 (5)
(b)	$r=\frac{1}{2}\Rightarrow a=\frac{150}{\frac{3}{4}}$ or $\frac{250}{1+\frac{1}{4}}=200$	M1A1
	$\frac{200\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\frac{1}{2}\right)}>399.99$ (Accept =)	M1A1ft
	$400\left(1-\left(\frac{1}{2}\right)^n\right)>399.99$ (Accept =)	
	$\left(\frac{1}{2}\right)^n<\frac{0.01}{400}$ $n>15.28...$ (Accept =)	M1
	All terms positive $\therefore$ least $n=16$	A1 (6) [11]

Question Number	Scheme	Marks
7(a)	$a^5 = 1024$	
	$a = \sqrt[5]{1024},\ a = 4$	B1 (1)
(b)	$6c + 9 = 3^4 = 81, \qquad c = 12$	M1, A1 (2)
(c)	$4\log_b 5 + 6\log_b 5 = 5$	M1
	$\log_b 5 = \frac{1}{2}$	M1
	$5 = b^{\frac{1}{2}} \quad b = 25$	M1A1 (4)
(d)	$3\log_2 x + 4\log_3 y = 10, \qquad 2\log_2 x - 4\log_3 y = 2$	
	$\Rightarrow 5\log_2 x = 12$,	M1A1
	$10\log_3 y = 7$	A1
	$x = 2^{2.4},\ \ x = 5.28$	M1A1
	$y = 3^{0.7},\ \ y = 2.16$	A1 (6)
		[13]

Question Number	Scheme	Marks
8 (a)	$AB=\sqrt{(13-1)^2+(7-1)^2}=\sqrt{180}\ \left(=6\sqrt{5}\right)$	M1A1 (2)
(b)	$x=5,\ y=5$ or $(5,5)$	B1B1 (2)
(c)	Grad $AB=\dfrac{-6}{12}=-\dfrac{1}{2}$	M1
	Grad perp $-1\div\left(-\dfrac{1}{2}\right)=2$	A1
	$y-5=2(x-5),\quad y=2x-5$	M1,A1 (4)
(d)	$x=9\quad y=d=13$	B1 (1)
(e)	E is $(7,9)$	B1
	Area $ADBE=\dfrac{1}{2}\begin{vmatrix}1 & 9 & 13 & 7 & 1\\ 7 & 13 & 1 & 9 & 7\end{vmatrix}$	M1A1ft
	$\dfrac{1}{2}\left(\left(1\times13+9\times1+13\times9+7\times7\right)-\left(1\times9+7\times1+13\times13+9\times7\right)\right)$	M1
	$=-30$	
	Area $=30$ units^2	A1 (5)
	ALT for (e):	
	$CD=4\sqrt{5}$	B1
	Area $=\Delta ADB-\Delta AEB,\quad =\dfrac{1}{2}AB\times\dfrac{1}{2}CD$	M1,A1
	Area $=\dfrac{1}{2}\times6\sqrt{5}\times2\sqrt{5},\ =30$	M1A1
		[14]

Question Number	Scheme	Marks
9 **(a)**	$\cos 2\theta = \cos^2\theta - \sin^2\theta$	
	$\cos 2\theta = \cos^2\theta - \left(1 - \cos^2\theta\right)$	M1
	$\cos^2\theta = \frac{1}{2}\left(\cos 2\theta + 1\right)$ (*)	A1 cso (2)
(b)	$\mathrm{f}\left(\theta\right) = 8\left(\frac{1}{2}\left(\cos 2\theta + 1\right)\right)^2 + 4\times\frac{1}{2}\left(\cos 2\theta + 1\right) - 5$	M1
	$= 2\left(\cos^2 2\theta + 2\cos 2\theta + 1\right) + 2\cos 2\theta + 2 - 5$	M1
	$= 2\times\frac{1}{2}\left(\cos 4\theta + 1\right) + 4\cos 2\theta + 2\cos 2\theta - 1$	M1
	$= \cos 4\theta + 6\cos 2\theta$ (*)	A1 cso (4)
(c)	$8\cos^4 x + 4\cos^2 x - 5 - 6\cos 2x = -0.5$	
	$\cos 4x + 6\cos 2x - 6\cos 2x = -0.5$	M1
	$\cos 4x = -0.5$	A1
	$4x = 120,\ 240,\ 480,\ 600$	M1
	$x = 30,\ 60,\ 120,\ 150$	A1 (4)
(d)(i)	$\int \mathrm{f}\left(\theta\right)\mathrm{d}\theta = \int\left(\cos 4\theta + 6\cos 2\theta\right)\mathrm{d}\theta$	M1
	$= \frac{1}{4}\sin 4\theta + 3\sin 2\theta\ \left(+c\right)$	A1
(ii)	$\int_0^{\frac{\pi}{3}} \mathrm{f}\left(\theta\right)\mathrm{d}\theta = \frac{1}{4}\sin\frac{4\pi}{3} + 3\sin\frac{2\pi}{3}$	M1
	$-\frac{1}{4}\times\frac{\sqrt{3}}{2} + 3\times\frac{\sqrt{3}}{2},\ = \frac{11\sqrt{3}}{8}$	A1,A1 (5)
		[15]

Question Number	Scheme	Marks
10 (a)	$x=\frac{1}{2}$	B1 (1)
(b)	$\frac{dy}{dx}=8-2(2x-1)^{-2}$	M1A1
	$2(2x-1)^{-2}=8$	
	$2x-1=\pm\frac{1}{2}\quad x=\frac{3}{4},\ x=\frac{1}{4}$ *	M1A1A1cso
	$\frac{d^2y}{dx^2}=8(2x-1)^{-3}$	M1A1
	Establish sign of $\frac{d^2y}{dx^2}$ at $x=\frac{3}{4},\quad x=\frac{1}{4}$	dM1
	$\therefore$ min at $x=\frac{3}{4}$ max at $x=\frac{1}{4}$	A1cso (9)
(c)	(i) $x=\frac{3}{4}\quad y=8$	B1
	(ii) $x=\frac{1}{4}\quad y=0$	B1
	(iii) $x=0\quad y=-1$	B1 (3)
(d)		B1 2 branches B1ft asymptote B1ft Max/min & y intercept (3) [16]

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