

Candidate surname

Other names

**Pearson Edexcel**  
International  
Advanced Level

Centre Number

Candidate Number

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**Wednesday 22 January 2020**

Morning (Time: 1 hour 30 minutes)

Paper Reference **WMA13/01**

## **Mathematics**

### **International Advanced Level**

#### **Pure Mathematics P3**

**You must have:**

Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations.**

**Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### **Instructions**

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  - *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### **Information**

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

### **Advice**

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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P60568A



1. A population of a rare species of toad is being studied.

The number of toads,  $N$ , in the population,  $t$  years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \quad t \geq 0, t \in \mathbb{R}$$

According to this model,

- (a) calculate the number of toads in the population at the start of the study, (1)

(b) find the value of  $t$  when there are 420 toads in the population, giving your answer to 2 decimal places. (4)

(c) Explain why, according to this model, the number of toads in the population can never reach 500 (1)

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## **Question 1 continued**

Q1

**(Total 6 marks)**

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*Turn over* 

2. The function  $f$  and the function  $g$  are defined by

$$f(x) = \frac{12}{x+1} \quad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2} \ln x \quad x > 0, x \in \mathbb{R}$$

- (a) Find, in simplest form, the value of  $fg(e^2)$  (2)

(b) Find  $f^{-1}$  (3)

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x) \quad (3)$$



**Question 2 continued**

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## **Question 2 continued**

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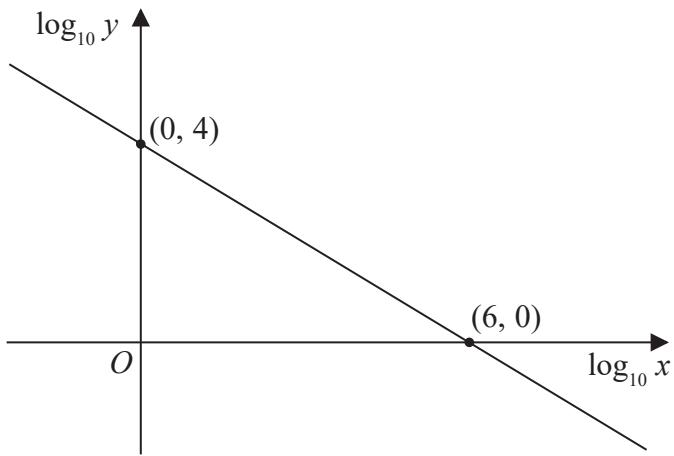
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Q2

**(Total 8 marks)**

3.



**Figure 1**

Figure 1 shows a linear relationship between  $\log_{10} y$  and  $\log_{10} x$

The line passes through the points  $(0, 4)$  and  $(6, 0)$  as shown.

- (a) Find an equation linking  $\log_{10} y$  with  $\log_{10} x$  (2)

(b) Hence, or otherwise, express  $y$  in the form  $px^q$ , where  $p$  and  $q$  are constants to be found. (3)



### **Question 3 continued**

Q3

**(Total 5 marks)**



$$4. \quad (i) \quad f(x) = \frac{(2x+5)^2}{x-3} \quad x \neq 3$$

- (a) Find  $f'(x)$  in the form  $\frac{P(x)}{Q(x)}$  where  $P(x)$  and  $Q(x)$  are fully factorised quadratic expressions.

(b) Hence find the range of values of  $x$  for which  $f(x)$  is increasing.

(6)

(ii)

$$g(x) = x \sqrt{\sin 4x} \quad 0 \leq x < \frac{\pi}{4}$$

The curve with equation  $y = g(x)$  has a maximum at the point  $M$ .

Show that the  $x$  coordinate of  $M$  satisfies the equation

$$\tan 4x + kx = 0$$

where  $k$  is a constant to be found.

(5)



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## **Question 4 continued**

Q4

**(Total 11 marks)**



5. (a) Use the substitution  $t = \tan x$  to show that the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

can be written in the form

$$5t^4 - 24t^2 - 5 = 0 \quad (4)$$

- (b) Hence solve, for  $0^\circ \leq x < 360^\circ$ , the equation

$$12 \tan 2x + 5 \cot x \sec^2 x = 0$$

Show each stage of your working and give your answers to one decimal place.

(4)

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### **Question 5 continued**

Q5

(Total 8 marks)



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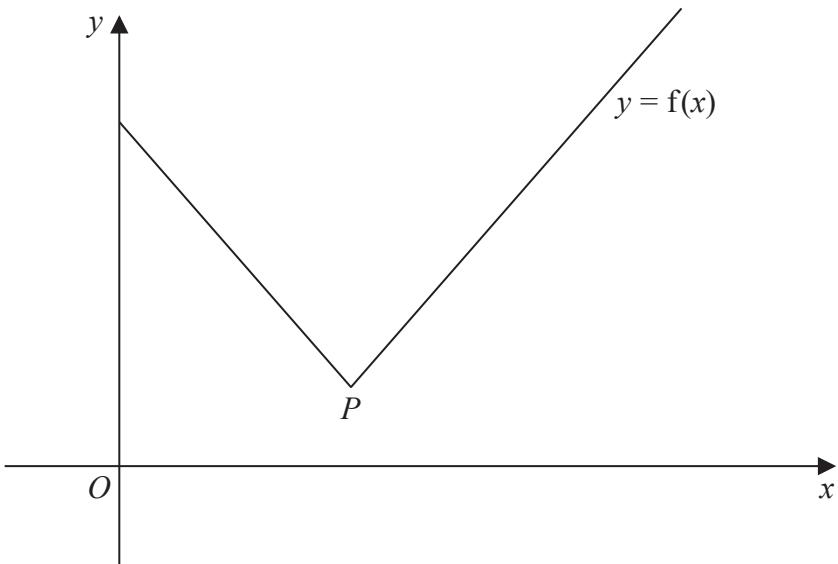
**Figure 2**

Figure 2 shows part of the graph with equation  $y = f(x)$ , where

$$f(x) = 2|2x - 5| + 3 \quad x \geq 0$$

The vertex of the graph is at point  $P$  as shown.

(a) State the coordinates of  $P$ .

(2)

(b) Solve the equation  $f(x) = 3x - 2$

(4)

Given that the equation

$$f(x) = kx + 2$$

where  $k$  is a constant, has exactly two roots,

(c) find the range of values of  $k$ .

(3)

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## **Question 6 continued**

Q6

(Total 9 marks)



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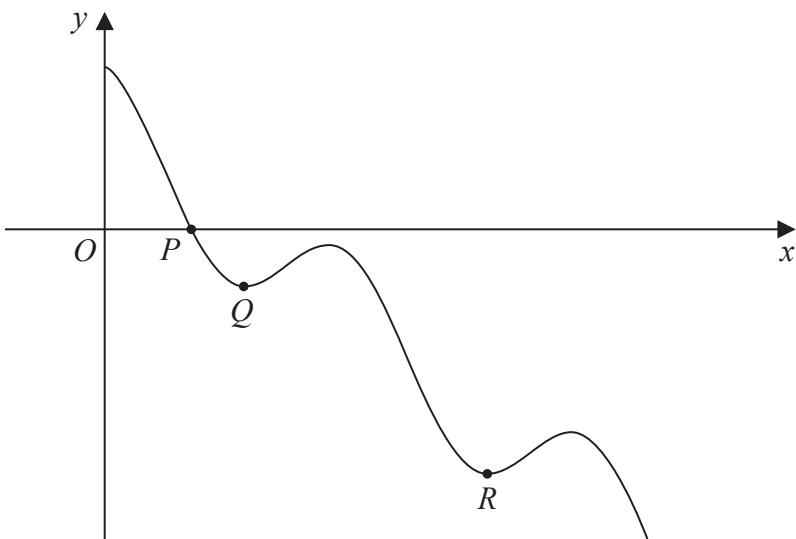
**Figure 3**

Figure 3 shows a sketch of part of the curve with equation

$$y = 2 \cos 3x - 3x + 4 \quad x > 0$$

where  $x$  is measured in radians.

The curve crosses the  $x$ -axis at the point  $P$ , as shown in Figure 3.

Given that the  $x$  coordinate of  $P$  is  $\alpha$ ,

- (a) show that  $\alpha$  lies between 0.8 and 0.9

(2)

The iteration formula

$$x_{n+1} = \frac{1}{3} \arccos(1.5x_n - 2)$$

can be used to find an approximate value for  $\alpha$ .

- (b) Using this iteration formula with  $x_1 = 0.8$  find, to 4 decimal places, the value of

(i)  $x_2$

(ii)  $x_5$

(3)

The point  $Q$  and the point  $R$  are local minimum points on the curve, as shown in Figure 3.

Given that the  $x$  coordinates of  $Q$  and  $R$  are  $\beta$  and  $\lambda$  respectively, and that they are the two smallest values of  $x$  at which local minima occur,

- (c) find, using calculus, the exact value of  $\beta$  and the exact value of  $\lambda$ .

(6)



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## Question 7 continued

Q7

**(Total 11 marks)**



8. (i) Find, using algebraic integration, the exact value of

$$\int_3^{42} \frac{2}{3x-1} dx$$

giving your answer in simplest form.

(4)

(ii) 
$$h(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \quad x > 1$$

Given  $h(x) = Ax + B + \frac{C}{(x-1)^2}$  where  $A$ ,  $B$  and  $C$  are constants to be found, find

$$\int h(x) dx$$

(6)

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## **Question 8 continued**

Q8

(Total 10 marks)



$$9. \quad f(\theta) = 5 \cos \theta - 4 \sin \theta \quad \theta \in \mathbb{R}$$

- (a) Express  $f(\theta)$  in the form  $R\cos(\theta + \alpha)$ , where  $R$  and  $\alpha$  are constants,  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ . Give the exact value of  $R$  and give the value of  $\alpha$ , in radians, to 3 decimal places.

(3)

The curve with equation  $y = \cos \theta$  is transformed onto the curve with equation  $y = f(\theta)$  by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

- (b) (i) describe fully the transformation that is a stretch,  
(ii) describe fully the transformation that is a translation.

(2)

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \quad \theta \in \mathbb{R}$$

- (c) find the range of  $g$ .

(2)



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## **Question 9 continued**

Q9

**(Total 7 marks)**

END

**TOTAL FOR PAPER IS 75 MARKS**

