



Mark Scheme (Results)

January 2019

Pearson Edexcel International Advanced Level In
Pure Mathematics P1 (WMA11/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - \square or d... The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	$\int \frac{2}{3}x^3 - \frac{1}{2x^3} + 5 dx = \frac{2}{3} \times \frac{x^4}{4} - \frac{1}{2} \times \frac{x^{-2}}{-2} + 5x + c$ $= \frac{1}{6}x^4 + \frac{1}{4}x^{-2} + 5x + c$	M1 A1 A1 A1 (4 marks)

M1 For raising any power by 1 eg. $x^3 \rightarrow x^4$, $x^{-3} \rightarrow x^{-2}$, $5 \rightarrow 5x$ or eg. $x^3 \rightarrow x^{3+1}$

A1 For two of $\frac{2}{3} \times \frac{x^4}{4}$, $-\frac{1}{2} \times \frac{x^{-2}}{-2}$, $+5x$ correct (un-simplified). Accept $5x^1$

This may be implied by a correct simplified answer

A1 For two of $\frac{1}{6}x^4$, $+\frac{1}{4}x^{-2}$, $+5x$ correct and in simplest form. Accept forms such as $\frac{x^4}{6}$, $\frac{1}{4x^2}$,

CONDONE $+\frac{0.25}{x^2}$ but NOT $\frac{1/4}{x^2}$, $\frac{5x}{1}$, $-\left(-\frac{1}{4}x^{-2}\right)$

A1 Fully correct and simplified with $+c$ all on one line. Accept simplified equivalents (see above) and ignore any spurious notation. ISW after a correct simplified answer is achieved.

A common mistake is writing $-\frac{1}{2x^3} = -2x^{-3} \rightarrow -x^{-2}$ this can still get the method mark for increasing the power by 1.

Question Number	Scheme	Marks
2.	<p>Attempts both sides as powers of 3 $\frac{3^x}{3^{4y}} = 3^3 \times 3^{0.5} \Rightarrow 3^{x-4y} = 3^{3.5}$</p> <p>Sets powers equal and attempts to makes y the subject : $x - 4y = 3.5 \Rightarrow y = \dots$</p> $y = \frac{1}{4}x - \frac{7}{8}$	<p>M1</p> <p>dM1</p> <p>A1</p> <p>(3) (3 marks)</p>
Alt1	<p>Multiplies by 3^{4y} first:</p> <p>Attempts both sides as powers of 3 $3^x = 27\sqrt{3} \times 3^{4y} \Rightarrow 3^x = 3^{3.5+4y}$ (Addition law on RHS)</p> <p>Sets powers equal and makes y the subject $x = 3.5 + 4y \Rightarrow y = \dots$</p> $y = \frac{1}{4}x - \frac{7}{8}$	<p>M1</p> <p>dM1</p> <p>A1</p>
Alt2	<p>Divides by $27\sqrt{3}$ first:</p> <p>Attempts both sides as powers of 3 $\frac{3^x}{3^{4y} \times 27\sqrt{3}} = 1 \Rightarrow 3^{x-4y-3.5} = 3^0$ (Subtraction law on LHS)</p> <p>Sets powers equal and makes y the subject $x - 3.5 - 4y = 0 \Rightarrow y = \dots$</p> $y = \frac{1}{4}x - \frac{7}{8}$	<p>M1</p> <p>dM1</p> <p>A1</p>
Alt3	<p>Takes logs of both sides</p> <p>Eg Base 3:</p> $\log_3 \left(\frac{3^x}{3^{4y}} \right) = \log_3 (27\sqrt{3})$ $\log_3 3^x - \log_3 3^{4y} = \log_3 27\sqrt{3}$ $x - 4y = 3.5 \Rightarrow y = \dots$ $y = \frac{1}{4}x - \frac{7}{8}$	<p>M1</p> <p>dM1</p> <p>A1</p>

M1 Attempts to use the subtraction law on the LHS and the addition law on the RHS to achieve a form of $3^m = 3^n$

Condone errors writing $27\sqrt{3}$ as a single power of 3 but it must be clear what the two indices are before adding if they make an error ($27 = 3^a$ and $\sqrt{3} = 3^b$ so $27\sqrt{3} = 3^{a+b}$)

A common mistake is to write $27\sqrt{3} \Rightarrow \sqrt{9} \times \sqrt{3} \times \sqrt{3} = 3^2$ which can be condoned and they can still get M1M1A0.

They may rearrange the equation first so look for attempts at the appropriate index laws being applied (see alternatives). In Alt2 allow the RHS=1.

They may use logs on both sides (the most likely would be base 3 - see Alt3) To score M1 they would need to take logs and then apply the laws of logs to either add or subtract.

dM1 Dependent upon the previous M mark, it is for an attempt to make y the subject. For this mark follow through their power for $27\sqrt{3}$ but they must have 3 terms in their equation relating to the powers and they cannot "lose" one in the rearrangement. (i.e $ax + by + c = 0$ or where $a, b, c \neq 0$) Do not award this mark if they rearrange to make x the subject. Condone sign slips only.

A1 $y = \frac{1}{4}x - \frac{7}{8}$ or exact simplified equivalent eg $y = \frac{2x-7}{8}$, $y = 0.25x - 0.875$

DO NOT ACCEPT $y = \frac{x - \frac{7}{2}}{4}$ or $y = \frac{x - 3.5}{4}$

Question Number	Scheme	Marks
3.(a)	Attempts to make y the subject States $-\frac{3}{5}$ or exact equivalent	M1 A1 (2)
(b)	Uses perpendicular gradients rule \Rightarrow gradient $l_2 = \frac{5}{3}$ Forms equation of l_2 using $(6,-2)$ $y+2 = \frac{5}{3}(x-6)$ $y = \frac{5}{3}x - 12$	M1 M1 A1 (3) (5 marks)
Alt1(a)	Eg Coordinates of two points on the line $(0,1.4)$ and $(1,0.8)$ Gradient = $\frac{0.8-1.4}{1-0}$ Gradient = -0.6	M1 A1

(a)

M1 For an attempt to rearrange $3x + 5y - 7 = 0$ and make y the subject.Expect to see $\pm 5y = \dots$ followed by $y = \dots$ or equivalent. $\frac{3}{5}$ on its own is M0.

Alternatively they may find two pairs of coordinates and find the gradient between those two points.

Allow one slip in calculating the coordinates and it must be clear that they are attempting $\frac{y_2 - y_1}{x_2 - x_1}$ A1 For stating $-\frac{3}{5}$ or exact equivalent in (a). A correct answer implies both marks and isw after a correct gradient is stated.. (The value of c does not need to be correct).Do not allow $y = -\frac{3}{5}x + \frac{7}{5}$ without some statement for ' m '. Do not allow $-\frac{3}{5}x$.

(b)

M1 Uses the perpendicular gradient rule following through on their gradient from (a).
If a gradient is not given, follow through on their 'm'

M1 For the equation of a straight line with a **changed** gradient using $(6, -2)$.

So if (a) was $-\frac{3}{5}$, then $(y+2) = \frac{3}{5}(x-6)$ would score this. At least one bracket must be correct. If the form $y = mx + c$ is used they must proceed as far as finding c . They must either have shown their gradient and $(6, -2)$ substituted into $y = mx + c$ and rearrange (maybe with errors) to find c or if they show no working then their c must be correct.

A1 $y = \frac{5}{3}x - 12$ Allow exact equivalent values for their constants eg $y = \frac{10}{6}x - \frac{36}{3}$, $y = 1.\dot{6}x - 12$,
 $y = \frac{5x}{3} - 12$ but do not allow equations such as $y = 1.67x - 12$. ISW after a correct equation in the correct form is found.

Question Number	Scheme	Marks
4.	<p>When ---- represents $<$ or $>$ and —— represents \leq or \geq</p> <p>Either $2y \leq x$ or $y \geq 2x - \frac{1}{8}x^2$</p> <p>$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \dots$ or $x \leq \dots$</p> <p>$x < 16, 2y \leq x$ and $y \geq 2x - \frac{1}{8}x^2$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>(3 marks)</p>
Alt1	<p>When ---- represents \leq or \geq and —— represents $<$ or $>$</p> <p>Either $2y < x$ or $y > 2x - \frac{1}{8}x^2$</p> <p>$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \dots$ or $x \leq \dots$</p> <p>$x \leq 16, 2y < x$ and $y > 2x - \frac{1}{8}x^2$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p>

B1 Sight of $2y \leq x$ or $y \geq 2x - \frac{1}{8}x^2$. Either inequality is sufficient for B1 and they may be written in an equivalent correct form (see NB below)

NB Inequalities cannot be in terms of R

M1 Attempts to find the upper bound for x to define R . Solves to find where the quadratic intersects the x -axis and then uses their value to write $x < \dots$ or $x \leq \dots$. Use general principles for solving a quadratic equation (page 5). They do not need to find or state $x = 0$ and ignore any lower bound eg $0 < x < \dots$

A1 $2y \leq x, y \geq 2x - \frac{1}{8}x^2$ and $x < 16$ (Allow $A \leq x < 16$ where $A \leq 12$).

Candidates may write more than one inequality for a particular boundary. In these cases mark the last one. Correct inequalities labelled on the graph are also acceptable, however, an inequality written below takes precedence.

NB You may see $y \leq \frac{x}{2}$ for $2y \leq x$ or even $2x - \frac{1}{8}x^2 \leq y \leq \frac{x}{2}$ oe

Alternatively, some candidates may express their inequalities involving a boundary for a dashed line using \leq or \geq and a boundary for a solid line using $<$ or $>$. It may not always be clear so mark positively. See Alt1

Question Number	Scheme	Marks
5. (a)	$\left(\frac{\pi}{2}, -1\right)$	B1 B1 (2)
(b)	Sine curve thro' (0,0) with max/min of ± 1 π 2π	M1 Fully correct A1
(c)	(i) 30 but follow through on $10 \times$ the number of their solutions $0 \rightarrow 2\pi$ (ii) 32	B1ft B1 (2) (2) (6 marks)

(a)

B1 Either coordinate correct. They may state the coordinates separately or condone the lack of brackets for this mark. (Accept the x -coordinate as 90° or awrt 1.57 radians for this mark) If only one coordinate is stated, it must be clear if it is the x or y coordinate.

B1 For $\left(\frac{\pi}{2}, -1\right)$ or $x = \frac{\pi}{2}, y = -1$ Allow $\left[\frac{\pi}{2}, -1\right]$

SC $\left(-1, \frac{\pi}{2}\right)$ B1B0 (coordinates the wrong way round)

(b)

M1 For a sketch of a sine curve with at least one cycle starting at (or going through) the origin with the same maximum/minimum y -values as the $\cos 2x$ curve.
Condone poor/incorrect period and poor symmetry
Condone turning points appearing V shaped for this mark. If drawn on a separate diagram the maximum and minimum must appear to be ± 1 according to their axes and a complete cycle must be in the positive domain. Condone slight inaccuracies of the amplitude of their sine curve.

A1 A correct sketch of $\sin x$ between $-\frac{\pi}{2}$ and 3π . Labelling where the graph crosses the x -axis is not required. Turning points must appear curved. If multiple attempts are drawn and it is not clear which is their final attempt then withhold the A1.



Do not accept linear looking graphs so unless it is a clear V shape at one maximum or minimum then allow any curvature at the turning points. As a guide the curve should not go diagonally across the square either side of turning points. See graph on the right showing what is not acceptable. Where the graph crosses the x -axis, it must be within half a square of the correct points

(c) (i)

B1ft 30 or follow through on $10 \times$ the number of their solutions between 0 and 2π (where 2π should be on the graph - see mark scheme for position). The question said hence or otherwise so they may get B1 for 30 even if their graph does not suggest that number of solutions.

(ii)

B1 32

Question Number	Scheme	Marks
6.(a)	$f'(x) = 5x^{\frac{3}{2}} - 40$ Attempts $5x^{\frac{3}{2}} - 40 = 0 \Rightarrow x^{\frac{3}{2}} = ..$ $x = 4$	M1A1 M1 A1 cao (4)
(b)	$f''(x) = \frac{15}{2}x^{\frac{1}{2}} = 5$ $\Rightarrow x^{\frac{1}{2}} = .. \Rightarrow x = ..^2$	$x = \frac{4}{9}$ M1 M1 A1 (3) (7 marks)

(a)

M1 For reducing the power by one on either x term ($x^{\frac{5}{2}} \rightarrow x^{\frac{3}{2}}$ or $-40x \rightarrow -40$)

A1 Correct (but may be unsimplified) $[f'(x)] = 5x^{\frac{3}{2}} - 40$

M1 Attempts to solve their $f'(x) = 0 \Rightarrow x^{\frac{3}{2}} = ..$ by making their $x^{\frac{3}{2}}$ the subject. Their $f''(x)$ must be a changed function. This can be implied by their final answer. If their $f'(x) = 0$ is of the form $f'(x) = Ax^B + Cx^D$ then this mark can be awarded for taking a factor out eg. $x^B(A + Cx^{D-B})$ and doing the same as above on the terms in their bracket.

A1 $x = 4$ cao (do not accept ± 4)

(b)

M1 For reducing the power by one on one of their terms in $f'(x)$ and setting their $f''(x) = 5$

M1 For a correct method leading to $x = ..$ from an equation of the form $Ax^{\frac{1}{2}} = 5$.

Eg for $\frac{15}{2}x^{\frac{1}{2}} = 5$ either makes $x^{\frac{1}{2}}$ the subject and squares or squares both sides and makes x the subject. This can be implied by their final answer. Do not allow slips on the power.

A1 $x = \frac{4}{9}$ or exact equivalent

The question states that solutions based entirely on graphical or numerical methods are not acceptable. Therefore if no differentiation is shown then this will score no marks.

Question Number	Scheme	Marks
7.(a)	Attempts $\frac{\sin \angle ACB}{6.5} = \frac{\sin 35}{4.7}$	M1
	$\angle ACB = \text{awrt}(52 \text{ or } 53)^\circ$ or $\text{awrt}(127 \text{ or } 128)^\circ$	A1
	$\angle ACB = 127.5^\circ$	A1
	(b)	Eg $\frac{(AC)}{\sin 17.5^\circ} = \frac{6.5}{\sin 127.5^\circ}$ or $\frac{4.7}{\sin 35^\circ}$
$\left[\frac{(CD)}{\sin 75^\circ} = \frac{4.7}{\sin 127.5^\circ} \Rightarrow (CD) = \dots \Rightarrow (AC) + (CD) \right] = \text{awrt } 8.2$		A1
Total length of wood = $8.1 + 6.5 + 4.7 + 4.7 = \text{awrt } 24.1$		A1 (3)
Alt1(a)	$\cos 35 = \frac{AC^2 + 6.5^2 - 4.7^2}{2 \times 6.5 \times AC} \Rightarrow AC^2 - 13 \cos(35)AC + 20.16 = 0 \Rightarrow AC = \dots$ $\cos \angle ACB = \frac{AC^2 + 4.7^2 - 6.5^2}{2 \times AC \times 4.7} \text{ oe}$	M1

(a)

M1 Uses the sine rule with the angles and sides in the correct positions.

Alternatively they may use the cosine rule on ACB and then solve the subsequent quadratic to find AC and then use the cosine rule again

A1 $\angle ACB = \text{awrt}(52 \text{ or } 53)^\circ$ or $\text{awrt}(127 \text{ or } 128)^\circ$

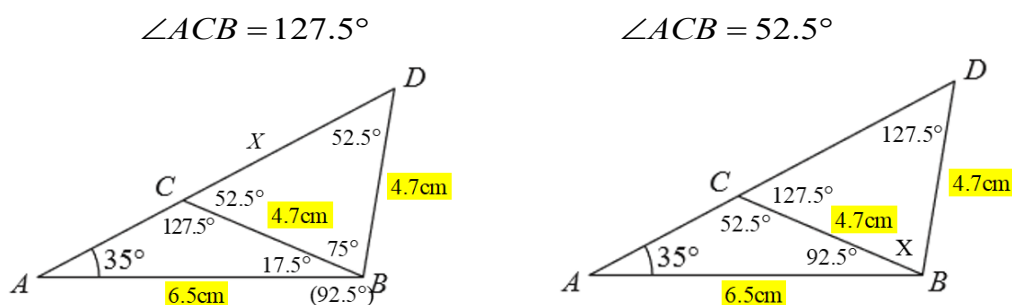
A1 $\angle ACB = 127.5^\circ$ only

(b) Working for (b) may be found in (a) which is acceptable

M1 Uses a formula that finds **part or all of the length AD** (eg AC , CD , AX , XD , AD).

The minimum required for this mark is the use of angle(s) and lengths in the correct places in the formula (which may have been rearranged to an alternative form). Condone mislabelling of the unknown length. This is usually the sine or cosine rule but they could split the triangle into two right angled triangles. See WAYS for additional guidance on methods. Sight of awrt8.2, awrt2.46 or awrt5.72 would imply this mark.

Condone angles in their triangles which do not add up to 180° and condone angles found with no working shown. For reference below these are the angles that would be found with $\angle ACB = 127.5^\circ$ and $\angle ACB = 52.5^\circ$ although they may “restart” so check their diagram as this may help.

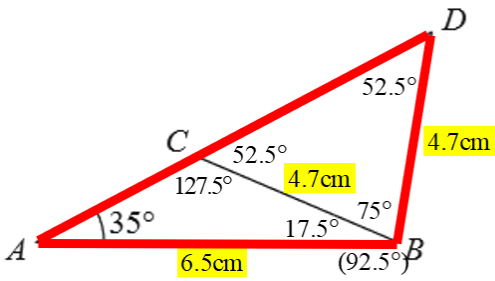
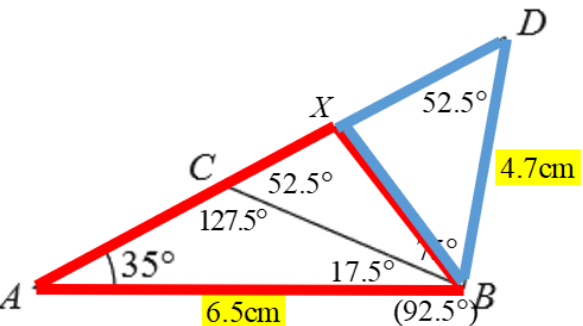


A1 awrt 8.2 Sight of awrt8.2 implies the length AD has been found. Ignore any labelling of lengths in their intermediate working and ignore any reference to AC (Accept "... = 8.2"). May be implied by a sum that totals awrt8.2 (eg awrt2.46+awrt5.72)

A1 awrt 24.1 ISW. Do not accept 24 or 25 (the length to the nearest metre) without seeing awrt24.1 or a calculation that totals awrt24.1 (eg $4.7 + 4.7 + 6.5 + 8.2 (= 24.1) \Rightarrow 24$)

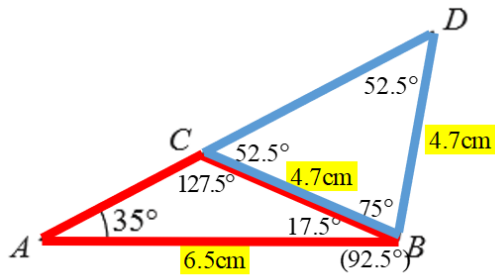
Candidates who assumed $\angle ACB = 52.5^\circ$ (acute) in (a):

Full marks can still be achieved as candidates may have restarted in (b) or not used the acute angle in their calculation which is often unclear. We are condoning any reference to $AC = 8.2$ so ignore any labelling of the lengths they are finding.

<p>WAY 1 (b)</p>	<p>Uses triangle ABD:</p>  <p>Option 1: Cosine rule</p> $4.7^2 + 6.5^2 - 2 \times 4.7 \times 6.5 \cos(180^\circ - 35^\circ - 52.5^\circ)$ <p>Option 2: Sine rule</p> $\frac{(AD)}{\sin 92.5^\circ} = \frac{4.7}{\sin 35^\circ} \text{ or } = \frac{6.5}{\sin 52.5^\circ}$ <p>Option 3: Cosine rule using 35° to form a quadratic</p> <p>Eg. $\cos 35^\circ = \frac{6.5^2 + (AD)^2 - 4.7^2}{2 \times 6.5 \times (AD)}$</p>	<p>M1</p> <p>M1</p> <p>M1</p>
<p>WAY 2 (b)</p>	<p>Forms two right angled triangles ABX and BDX :</p>  <p>Either of $6.5 \cos 35^\circ$ or $4.7 \cos 52.5^\circ$</p>	<p>M1</p>

WAY 3
(b)

Uses the triangles ABC and BCD :



Length AC

Option 1: Cosine rule

$$(AC)^2 = 4.7^2 + 6.5^2 - 2 \times 4.7 \times 6.5 \cos(180^\circ - 35^\circ - 127.5^\circ)$$

M1

Option 2: Sine rule

$$\frac{(AC)}{\sin 17.5^\circ} = \frac{6.5}{\sin 127.5^\circ} \text{ or } \frac{4.7}{\sin 35^\circ}$$

M1

Option 3: Cosine rule using 35° to form a quadratic in AC

$$\cos 35^\circ = \frac{6.5^2 + (AC)^2 - 4.7^2}{2 \times 6.5 \times (AC)}$$

M1

Length CD

Option 1: Cosine rule

$$(CD)^2 = 4.7^2 + 4.7^2 - 2 \times 4.7 \times 4.7 \cos(180^\circ - 2 \times 52.5^\circ)$$

Option 2: Sine rule

$$\frac{(CD)}{\sin 75^\circ} = \frac{4.7}{\sin 127.5^\circ}$$

M1

Option 3: Cosine rule using 52.5° to produce a quadratic in CD

$$\cos 52.5^\circ = \frac{4.7^2 + (CD)^2 - 4.7^2}{2 \times 4.7 \times (CD)}$$

M1

Option 4: Using isosceles triangle property on BCD to find half of CD

$4.7 \cos 52.5^\circ$ (they do not have to double to find CD for M1)

M1

Question Number	Scheme	Marks
8 (a)	States $y = 4$	B1 (1)
(b)	States (16,9) only	B1 (1)
(c)	$k \leq 4, k = 9$	B1, B1 (2)
(d) (i)	$a = 6$	B1 (2)
(ii)	$y = f(x - 3)$	B1 (2)
		(6 marks)

(a)

B1 $y = 4$ only. May be written on a new graph

(b)

B1 (16,9) condone lack of brackets and must be the only answer (or clearly their final answer). May be written on a graph again.

(c)

B1 Sight of either one of $k \leq 4, k = 9$ Must be in terms of k . Ignore any others for this markB1 Both of $k \leq 4, k = 9$ ONLY Where several inequalities or equations are given only mark what appears to be their final answer for the $k = \dots$ and their final answer for $k \leq \dots$ Ignore any inequalities which are within $k \leq 4$ SC $y \leq 4, y = 9$ (They never write in terms of k so score B1B0)

(d)(i)

B1 $a = 6$ (6 on its own is sufficient). Also allow $(y=) f(x) - 6$ and isw if they proceed to state $a = -6$ B1 $y = f(x - 3)$ Also accept $y = -f(x) + 6, y = f(x + 4) - 9$ or even rearrangements such as $f(x) = 6 - y$ Do not accept combinations of different transformations as the question asked for a single transformation.

Question Number	Scheme	Marks
9.	$\frac{3}{x} + 5 = -2x + c$	
	Multiplies through by x $3 + 5x = -2x^2 + cx \Rightarrow \pm 2x^2 \dots\dots (= 0)$	M1
	and writes in quadratic form $\Rightarrow 2x^2 + (5 - c)x + 3 (= 0)$ oe	A1
	Attempts " $b^2 - 4ac = (5 - c)^2 - 24$ "	M1
	Attempts " $b^2 - 4ac = 0 \Rightarrow (5 - c)^2 - 24 = 0 \Rightarrow c = \dots$ "	dM1
	$(c =) 5 \pm 2\sqrt{6}$ oe	A1
	Attempt at inside region $5 - 2\sqrt{6} < c < 5 + 2\sqrt{6}$ oe	M1 A1
		(7) (7 marks)

- M1 Attempts to multiply through by x and moves all terms to one side. This may be implied by later work. Condone one term not being multiplied by x for this mark but all four terms must be on one side. You do not have to see the " $=0$ ".
- A1 Correct quadratic with the terms in x factorised, or correct values of a , b and c . Accept the equivalent form where terms have been collected on the other side. The " $=0$ " is not needed for this mark and ignore the use of any inequalities.
- M1 Attempts $b^2 - 4ac$ using their values. You may see " $b^2 \dots 4ac$ ". It is sufficient to see the values substituted in correctly for this mark and you can condone invisible brackets. They must have achieved a quadratic in x to calculate the discriminant so do not allow if eg a is their coefficient of x^{-1} but do allow this mark if they have a quadratic from incorrect working.
- dM1 Attempts to solve their $b^2 - 4ac = 0$ to find at least one of the critical values for c . Must have achieved a quadratic in c . Apply general marking principles for solving a quadratic. This is dependent on the previous method mark and may be implied by solutions correct to 1dp for their quadratic.
- A1 Correct critical values $(c =) 5 \pm \sqrt{24}$ or exact equivalent of the form $\frac{a \pm \sqrt{b}}{c}$ (the critical values may appear within inequalities)
- M1 Finds inside region for their critical values. They may draw a diagram but they must proceed to an $\dots < c < \dots$ (allow use of \leq for one or both inequalities for this mark and they may even be separate statements) May be in terms of x or any other variable for this mark.

A1 $5 - 2\sqrt{6} < c < 5 + 2\sqrt{6}$ Must be in terms of c and must be exact

Accept others such as $5 - \sqrt{24} < c < 5 + \sqrt{24}$, $c > 5 - 2\sqrt{6}$ AND $c < 5 + 2\sqrt{6}$ $c > 5 - 2\sqrt{6}$, $c < 5 + 2\sqrt{6}$
 $5 + 2\sqrt{6} > c > 5 - 2\sqrt{6}$, $\{c : 5 - 2\sqrt{6} < c < 5 + 2\sqrt{6}\}$, $\frac{10 - \sqrt{96}}{2} < c < \frac{10 + \sqrt{96}}{2}$

Do NOT accept $c > 5 - 2\sqrt{6}$ OR $c < 5 + 2\sqrt{6}$

Question Number	Scheme	Marks
10. (a)	Correct equations $\frac{1}{2}r^2\theta = 6, \quad 2r + r\theta = 10$	B1 B1
	Eliminates $r = \frac{10}{2+\theta} \Rightarrow \frac{1}{2}\left(\frac{10}{2+\theta}\right)^2 \theta = 6$ $\Rightarrow 50\theta = 6(4 + 4\theta + \theta^2) \Rightarrow 3\theta^2 - 13\theta + 12 = 0 \quad *$	M1 A1* (4)
(b)	$(3\theta - 4)(\theta - 3) = 0 \Rightarrow \theta = \frac{4}{3}, 3$	B1
	$\theta = \frac{4}{3}, r = 3 \quad \theta = 3, r = 2$	M1 A1 (3) (7 marks)

(a)

B1 One correct equation $\frac{1}{2}r^2\theta = 6$ or $2r + r\theta = 10$ (may use φ instead of θ which is fine)

B1 Two correct equations $\frac{1}{2}r^2\theta = 6$ and $2r + r\theta = 10$ ($r + r + r\theta = 10$ is acceptable)

Note you may see one of both of these equations in (b) which you can award the marks retrospectively. They may be implied from later work:

Eg. $r^2 = \frac{12}{\theta} \Rightarrow 2\sqrt{\frac{12}{\theta}} + \sqrt{\frac{12}{\theta}}\theta = 10$ implies B1B1

M1 Scored for eliminating r and reaching an equation in θ only. The initial equations must be of a similar form to the area of the sector and the perimeter but possibly with errors (look for $\dots r^2\theta = 6$ and $2r + \dots r\theta = 10$). Condone errors when rearranging but their subsequent substitution must be correct for their rearrangement. Eg. $10 = r\theta + 2r \Rightarrow \frac{10}{\theta} = 3r$

A1* Reaches the given answer with no errors. There must be at least one intermediate line of manipulation following the elimination stage and $(\theta + 2)^2$ must be multiplied out correctly before achieving the final answer to score full marks.

(b)

B1 $\theta = \frac{4}{3}, 3$ (They may just appear from a calculator)

M1 Substitutes one of their values of θ into one of the previous equations of the allowable form and proceeds as far as $r = \dots$

A1 Both results $\theta = \frac{4}{3}, r = 3$ $\theta = 3, r = 2$

Values must be exact. Withhold the final mark if they do not rule out $r = -3 / r = -2$ from using the area of a sector equation and withhold the final mark if the corresponding values of r are incorrectly paired to the other values of θ ($\theta = \frac{4}{3}, r = 2$ and $\theta = 3, r = 3$)

Question Number	Scheme	Marks
11. (a)	<p style="text-align: right;">∩ shaped quadratic</p> <p style="text-align: right;">Intercepts at O and 3</p> <p style="text-align: center;">2 3 5</p> <p style="text-align: right;">-ve cubic</p> <p style="text-align: right;">Intercepts at O, 2 and 5</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p style="text-align: right;">(4)</p>
(b)	<p>Sets $x(x-2)(5-x) = x(3-x)$</p> <p>$3x - x^2 = -x^3 + 7x^2 - 10x \Rightarrow \pm(x^3 - 8x^2 + 13x) (=0)$ OR</p> <p>$\pm x\{(x-2)(5-x) - (3-x)\} (=0)$</p> <p>Proceeds to $x(x^2 - 8x + 13) = 0$ *</p>	<p>M1</p> <p>dM1</p> <p>A1*</p> <p style="text-align: right;">(3)</p>
(c)	<p>Solves $x^2 - 8x + 13 = 0 \Rightarrow x = 4 \pm \sqrt{3}$</p> <p>Substitutes $x = "4 - \sqrt{3}"$ into $y = x(3-x)$ oe</p> <p>$y = (4 - \sqrt{3})(-1 + \sqrt{3}) = -4 + 4\sqrt{3} + \sqrt{3} - 3 = \dots$</p> <p>$y = -7 + 5\sqrt{3}$</p>	<p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">(12 marks)</p>

(a) For both parts they must have a graph

(i)

B1 ∩ shaped quadratic appearing anywhere on the graph. This is for the general shape so do not be concerned with any parts which appear linear.

B1 A quadratic which crosses the x -axis at O and 3. Accept a mark of 3 on the x -axis. The origin does not need to be labelled as a point of intersection.

(ii)

B1 -ve cubic appearing anywhere on the graph with a maximum and a minimum. This is for the general shape so do not be concerned with any parts which appear linear.

B1 A cubic which crosses the x -axis at O , 2 and 5. Accept 2 and 5 marked on the x -axis. The origin does not need to be labelled as a point of intersection.

It is not a requirement that the curves meet in quadrant 4. The relative heights of the maximum points are not important either. Points of intersection with the coordinate axes may be listed separately. Condone the lack of brackets but the x and y coordinates must be the correct way round. They cannot simply state eg $x = 2, x = 5$

(b)

M1 Sets the equations equal to each other

dM1 Multiplies out and collects terms on one side (unsimplified). Condone errors in multiplying out and slips in collecting like terms but x must be a factor of each term. Condone invisible brackets for this mark and condone the absence of “=0”

Alternatively takes the factorised forms to one side and factors out the x term.

A1* Proceeds to $x(x^2 - 8x + 13) = 0$ with no errors including brackets. As a minimum you must see $(x - 2)(5 - x)$ being multiplied out, terms being collected on one side and a factor of x being taken out.

(c)

M1 Solves $x^2 - 8x + 13 = 0$ by completing the square or formula. Apply general marking principles for quadratics for this mark. Their solutions do not need to be exact for this mark.

A1 Either (or both) of $(x =) 4 \pm \sqrt{3}$ (oe but must be of the form $\frac{a \pm \sqrt{b}}{c}$ or simplified further)

M1 Substitutes $x = 4 - \sqrt{3}$ or their **lower** value of x into either equation to find y . This mark can be awarded if they had rounded decimal solutions to their quadratic. If they substitute both values in then this mark can still be awarded. You may need to check their y value on a calculator to imply this method mark if their x value is incorrect.

M1 Evidence of using the rules of surds to form a y coordinate that is exact and simplified. They must show evidence of working with surds before simplifying to two terms of the form $d + f\sqrt{g}$. Eg one of the bold terms from $y = (a - \sqrt{b})(c + \sqrt{b}) = ac + \mathbf{a}\sqrt{\mathbf{b}} - \mathbf{c}\sqrt{\mathbf{b}} - \mathbf{b}$ would be sufficient. If no working shown then M0 and A0 will follow this.

A1 $(4 - \sqrt{3}, -7 + 5\sqrt{3})$ or exact equivalent. This must be the only coordinate stated as their final answer.

Note that the question stated using algebra and showing your working. If they simply state the solutions of the quadratic then they can only get M0A0M1M1A0.

If they simply state the solutions of the quadratic and show no surd work then the maximum they may be able to get is M0A0M1M0A0.

Question Number	Scheme	Marks
12.(a)	Substitutes $x = 4$ in $\frac{dy}{dx} = 3x\sqrt{x} - 10x^{-\frac{1}{2}} = 3 \times 4 \times 2 - \frac{10}{2} = 19$ Attempts $(y - (-2)) = "19" \times (x - 4) \Rightarrow y = 19x - 78$	M1A1 M1A1 cao (4)
(b)	$f'(x) = 3x^{\frac{3}{2}} - 10x^{-\frac{1}{2}} \Rightarrow f(x) = \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} + c$ $x = 4, f(x) = -2 \Rightarrow$ $-2 = 38.4 - 40 + c \Rightarrow c = \dots(-0.4)$ $[f(x) =] \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} - 0.4$	M1 A1 A1 M1 A1 cso (5) (9 marks)

(a)

M1 Substitutes $x = 4$ into $\frac{dy}{dx} = 3x\sqrt{x} - 10x^{-\frac{1}{2}}$. Do not award this mark if they attempt to differentiate the expression first (look at the $-10x^{-\frac{1}{2}}$ for evidence of the power decreasing) but do condone an error made on the power of the first x term if they try to write it as a single power of x .

A1 Gradient = 19

M1 Attempts an equation of a tangent using their $f'(4)$ and $(4, -2)$. If they attempt $(y + 2) = "19"(x - 4)$ at least one of the brackets must be correct. If the form $y = mx + c$ is used they must proceed as far as finding c . They must either have shown their gradient and $(4, -2)$ substituted into $y = mx + c$ and rearrange (maybe with errors) to find c or if they show no working then their c must be correct.

A1 $y = 19x - 78$ cao

(b)

M1 Raises the power of any term by one $x^{-\frac{1}{2}} \rightarrow x^{\frac{1}{2}}, x^{\frac{3}{2}} \rightarrow x^{\frac{5}{2}}$ Accept eg $x^{\frac{3}{2}} \rightarrow x^{\frac{3}{2}+1}$

A1 Any term correct (may be un-simplified) with or without $+c$

A1 Both terms correct (may be un-simplified) with or without $+c$

M1 Substitutes $x = 4, y = -2$ into their $f(x)$ containing $+c$ to obtain c . Condone errors in evaluating and rearranging

A1 $[f(x) =] \frac{6}{5}x^{\frac{5}{2}} - 20x^{\frac{1}{2}} - 0.4$ or equivalent including $(y =) \dots$ cso

